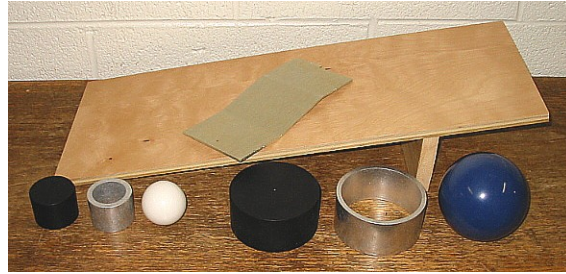


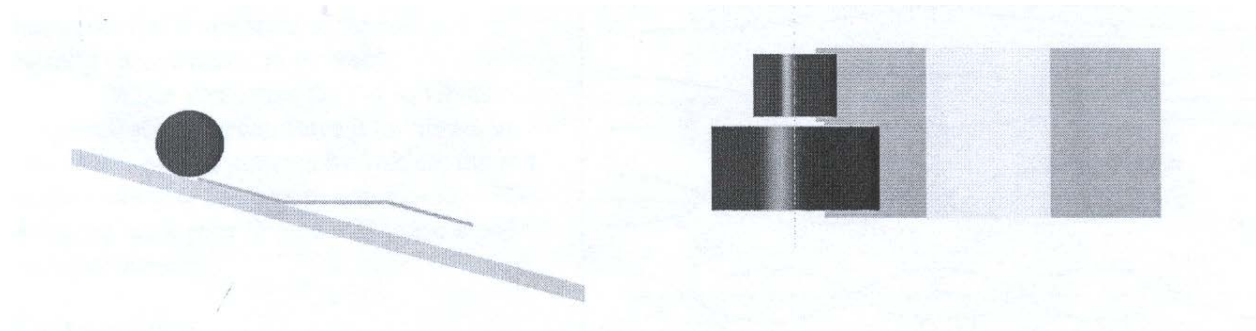
Moments of Inertia



Purpose: Demonstrates (in a fun way) how the distribution of mass in an object affects its rotational acceleration.

This is a classic demo in which the students try to guess which of the objects will roll down the inclined plane the fastest. One could make up an entire lecture out of this (I did). With the Pasco set, we have two sets of three objects (sphere, disk, and hoop): one set of radius $R = 5$ cm. and the other $R = 2.5$ cm.

The set comes with a tool to aid in starting the races: One edge is straight to hold and start objects of the same radius. The other side is stepped, so that objects of different radii are started with their centers aligned.



Note: I have gotten complaints that the objects do not all have the same mass, so that the demo is flawed from a philosophical point of view. Maybe so, but just try making the apparatus! (Pasco cannot!) To overcome this problem, the logical flow of the demo should go like this:

A) Start by racing objects of the same shape. This will (at least) suggest that the motion is independent of both the mass and radius. (Note the exception mentioned below about the hoops.)

B) Derive these facts for the class, e.g. the argument given below. Only the shape counts.

C) Then race objects of different shape against one another.

D) Finally, reexamine the behavior of the hoops.

Because the objects do not slide down the plane, there must be a tangential force $Mg\sin q_{in}$ (where q_{in} is the incline of the plane) acting on the point of contact with the plane.

The torque about the object's center is $RMg\sin q_{in}$. Since $I = fMR^2$ [with $f = 1$ (hoop), $1/2$ (disk), or $2/5$ (solid sphere)], the angular acceleration is independent of the object's mass:

$$a = t / I = g\sin q_{in} / f R.$$

So, for a given radius the sphere will accelerate down the plane the fastest. It has the least inertia since most of its mass is concentrated near the axis of rotation.

To compare objects of different radii, one must figure the distance $d = Rq$ the rolling object travels down the plane. Since $q = \frac{1}{2} at^2$, the time taken is

$$t = [2df / g \sin q_{in}]^{1/2} ,$$

independent of the radius, e.g. the large and small sphere should take the same time to roll down the plane. The smaller sphere has a larger a , but must revolve more to travel the same distance.

An interesting footnote: The large and small spheres and disks do in fact roll down together, however the hoops do not: The smaller hoop wins! This is due to the hoops having a finite thickness T – which is the same for both hoops. So while they look similar, they do not have the same ‘shape’.

From the formula for I of a thick-walled cylinder you can write $f = [1 - T/R + \frac{1}{2} (T/R)^2]$.

Assuming the same thickness for both hoops, you then expect the smaller hoop to have the smaller f , and so rolls down the quickest. Using the actual measurements you get $f = 0.77$ (small hoop) and $f = 0.90$ (large hoop).

Extra Equipment: None

Location: Shelf B2

See Manual