

## Torque and Angular Momentum

**Purpose:** Using a toy gyroscope on a good stand, it is fun to investigate the precession arising from the torque due to gravity.

Illustrates the concept of  $\vec{\tau} = d\vec{L}/dt$

This toy gyroscope is small but, with the help of its rack-and-gear startup strip, works very well. Since the fun comes from letting it go over until it drops - which is past horizontal - you would not want it any bigger.

The things to notice and ponder: 1) It precesses in different directions depending on which way it is spinning. 2) As friction robs energy, the axis tilts more and more horizontally, and the rate of precession increases.

**Note:** As we all know, tops can be pretty complicated, but the basic features for 1st year students can be explained as follows:

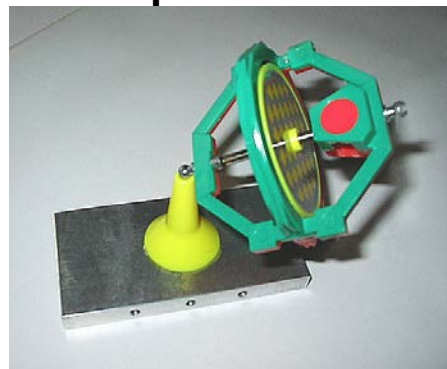
By leaning over, there is a torque about the anchor point due to gravity  $\tau = \mathbf{r}_{cm} \times M\mathbf{g}$ , which points perpendicularly to the left of the axial component  $\mathbf{r}_r$  of  $\mathbf{r}_{cm}$  (as seen from above). Because the top is fixed, only the axial component of  $\mathbf{L}$  can change: depending on which way the top is spinning,  $\mathbf{L}_r$  is either parallel or antiparallel to  $\mathbf{r}_r$ . Since  $D\mathbf{L}_r = \tau Dt$ , the top's axis must precess counterclockwise if the gyro is spinning up (counterclockwise) and clockwise if the top is spinning down (clockwise) – all as seen from above. (The diagram is easier than words.)

Why does the rate of precession increase as it tilts? One is tempted to glibly say that it is because the torque increases with angle. However the magnitude of  $L_r$  also increases with tilt. These cancel, and the precession rate is independent of the tilt:  $\omega_P = mgr/L$ .

I believe what is happening is that, as friction causes the wheel to lose speed,  $L$  decreases and so  $\omega_P$  increases. To test this out you can spin the gyro up and release it tilted over at the horizontal; it precesses at about the same rate as it does when started with a more modest tilt.

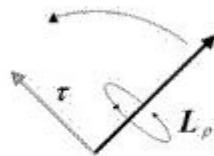


**Top on stand**

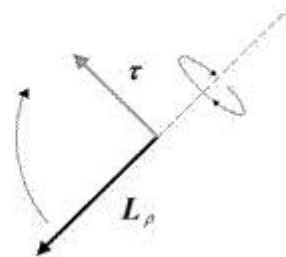


**The end is near!**

Spin up



Spin down



**As viewed from above.**

**Extra Equipment:** None

**Location:** Shelf A3

**Furthermore:** You can also show precession using the bike wheel shown in the Angular Momentum demo. By spinning it fast (use a cord) you can suspend it by the loop on the axle's end. The wheel holds itself horizontally and precesses like the top.

For those of you who want to demonstrate more complicated aspects of the gyroscope, we also have the large, motor-driven, gimbaled model shown below. (The weights for it are missing.) Located also on Shelf A3.

