

Tower of Lire

Purpose: Demonstrates the idea of the center of mass and the stability of structures.

Pile the boards on a table edge. By stacking them with the correct displacements, the stack can extend out over the table edge without falling.

The six 2×4s are labeled 1-6 (top to bottom), with lines drawn to indicate the edge points. If you can find some meter sticks, they work nicely. Particularly since, once the tower is built you can make a cool mobile-like object by rotating the objects around their balance points.

Note: This is a nice example of how the center-of-mass concept is useful in understanding the stability of structures.

The idea is easy to understand from the picture: Consider a stack of identical boards of length L .

For maximum extension, one starts with the top board's C.M. placed at the edge of the next board down. The C.M of these first two boards is thus $(1/2 + 1/4)L$ from the stack's end.

We can place this point on the end of the next board down, and treat it as a point mass of 2 units. The C.M of the three boards is then $L/6$ further to the right.

Continuing in this way, one finds that the maximum displacement for the n th board is $L/2n$.

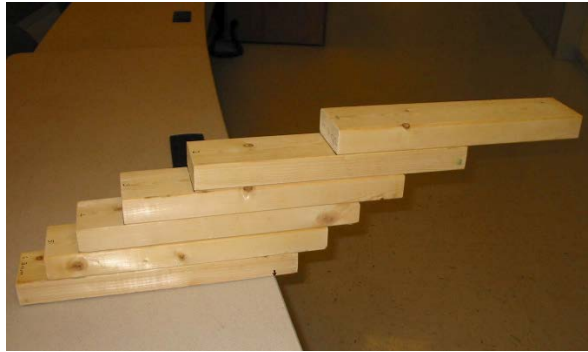
The total overhang for N boards is then

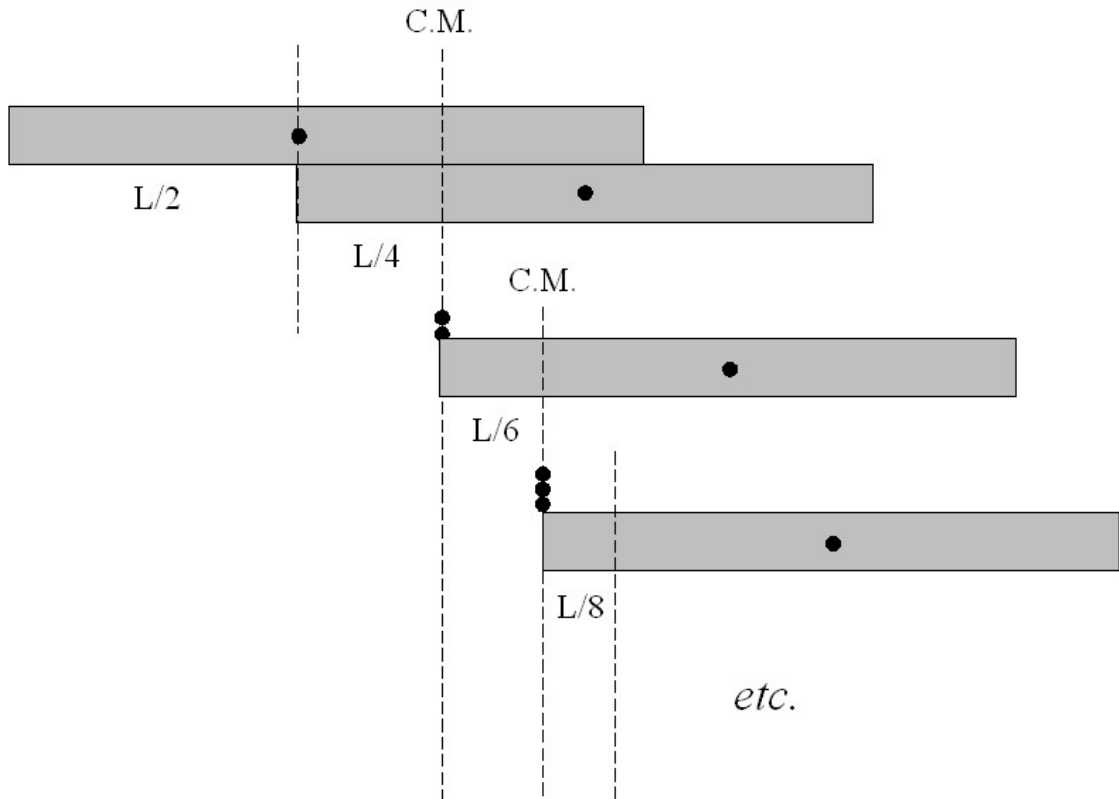
$$\text{overhang} = \frac{L}{2} \sum_{n=1}^N \left(\frac{1}{n}\right).$$

This series does not converge, so in theory you can make the overhang as large as you like by stacking more and more boards. It is fun to ask the class how many boards it would take to make the overhang so many times the length L . Here's a table:

N	Overhang/L
1	0.50
2	0.75
3	0.92
4	1.04
5	1.14
6	1.22
7	1.30
10	1.46
30	2.00
226	3.00
1673	4.00

Various Possibilities





Extra Equipment: None

Location: Shelf B2