A Primer in Optical Soliton Theory

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Introduction

The advent of fiber optics has revolutionized telecommunication systems around the world, enabling an unprecedented amount of information exchange, all at the speed of light. However, we are just at the beginning of what will likely be known as the photonics century. Just as electronics dramatically improved the quality of life in the 20th century, photonics promises to do the same in the 21st century.

One of the keys to the success of the ensuing photonics revolution will be the use of optical solitons in fiber optic communication systems. Solitons are a special breed of optical pulses that can propagate through an optical fiber undistorted for tens of thousands of kilometers. The key to soliton formation is the careful balance of the opposing forces of chromatic dispersion and self-phase modulation.

In this paper, the origin of optical solitons will be discussed starting with the basic concepts of optical pulse propagation. In order to maximize clarity and provide the reader with an intuitive physical understanding of the formation of solitons, the complicated mathematics behind this subject will be kept hidden. Readers interested in a more in-depth mathematical analysis of soliton theory should check out the references listed at the end of this paper.

Chromatic Dispersion

An electromagnetic wave, such as the light sent through an optical fiber, is actually a combination of electric and magnetic fields oscillating perpendicular to each other. When an electromagnetic wave propagates through free space, it travels at the constant speed of 3.0×10^8 meters per second, equivalent to about seven trips around the earth every second.

However, when light propagates through a material rather than through free space, the electric and magnetic fields of the light induce a polarization in the electron clouds of the material. This polarization makes it more difficult for the light to travel through the material, so the light must slow down to a speed less than its original 3.0×10^8 meters per second. The degree to which the light is slowed down is given by the material's refractive index *n*. The speed of light in within material is then

$$v = \frac{3.0 \times 10^8 \text{ meters per second}}{n}.$$

This equation shows that a high refractive index means a slow light propagation speed. Higher refractive indices generally occur in materials with higher densities, since a high density implies a high concentration of electron clouds to slow the light.

Since the interaction of the light with the material depends on the frequency of the propagating light, the refractive index is also dependent on the light frequency. This, in turn, dictates that the speed of light in the material depends on the light's frequency, a phenomenon known as chromatic dispersion.

To understand what chromatic dispersion means for fiber optic communication systems, one must first understand the nature of an optical pulse. Although an optical pulse represents only a single bit of information in a communication system, it is actually composed of many hundreds or even thousands of particles of light known as photons.

Optical pulses are often characterized by their shape, and a typical pulse shape is Gaussian, like that shown in Figure 1. In a Gaussian pulse, the constituent photons are concentrated toward the center of the pulse, making it more intense than the outer tails.

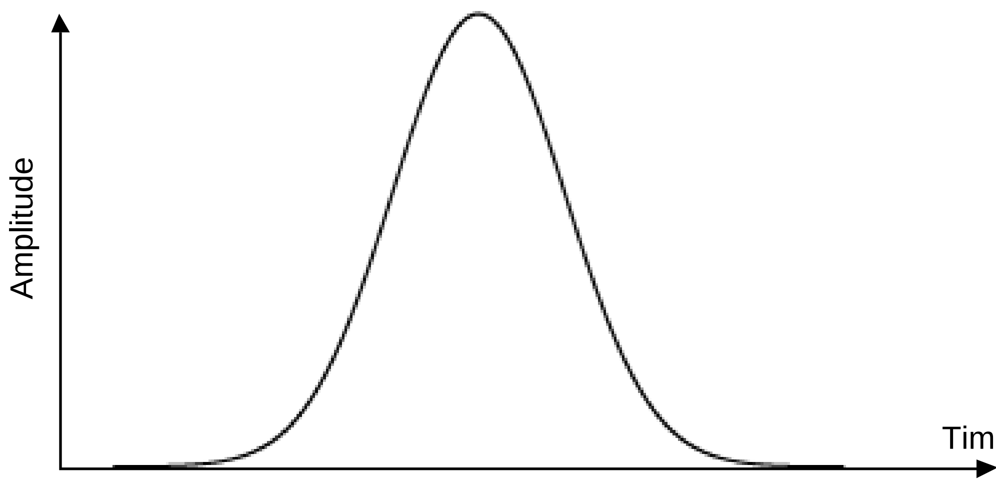

Figure 1   A Gaussian pulse

Optical pulses are generated by a near-monochromatic light source such as a laser or an LED. If the light source were completely monochromatic, then it would generate photons at a single frequency only, and all of the photons would travel through the fiber at the same speed. In reality, small thermal fluctuations and quantum uncertainties prevent any light source from being truly monochromatic. This means that the photons in an optical pulse actually include a range of different frequencies. Since the speed of a photon in an optical fiber depends on its frequency, the photons within a pulse will travel at slightly different speeds from each other. The result is that the slower photons will lag further and further behind the faster photons, and the pulse will broaden. An example of this broadening is given in Figure 2.

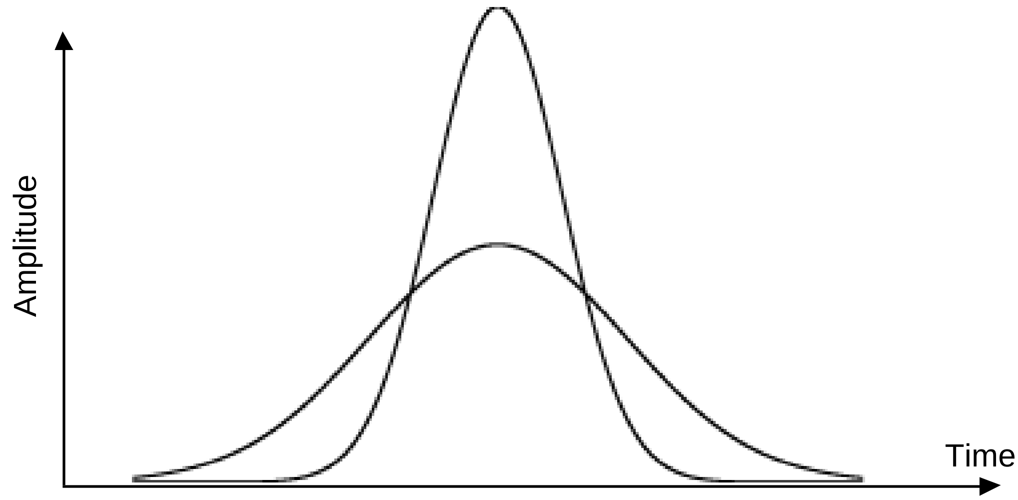

Figure 2 A pulse broadens due to chromatic dispersion

Chromatic dispersion may be classified into two different regimes: normal and anomalous. With normal dispersion, the lower frequency components of an optical pulse travel faster than the higher frequency components. The opposite is true with anomalous dispersion. The type of dispersion a pulse experiences depends on its wavelength; a typical fiber optic communication system uses a pulse wavelength of 1.55 μm, which falls within the anomalous dispersion regime of most optical fiber.

Pulse broadening, and hence chromatic dispersion, can be a major problem in fiber optic communication systems for obvious reasons. A broadened pulse has a much lower peak intensity than the initial pulse launched into the fiber, making it more difficult to detect. Worse yet, the broadening of two neighboring pulses may cause them to overlap, leading to errors at the receiving end of the system.

However, chromatic dispersion is not always a harmful occurrence. As we shall soon see, when combined with self-phase modulation, chromatic dispersion in the anomalous regime may lead to the formation of optical solitons.

Self-Phase Modulation

We saw in the previous section that a material's refractive index is dependent on the frequency of the light traveling through it. In fact, the refractive index is also dependent on the intensity of the light. This is due to the fact that the induced electron cloud polarization in a material is not actually a linear function of the light intensity. The degree of polarization increases nonlinearly with light intensity, so the material exerts greater slowing forces on more intense light. The result is that the refractive index of a material increases with the increasing light intensity. Phenomenological consequences of this intensity dependence of refractive index in fiber optics are known as fiber nonlinearities.

There exist many different types of fiber nonlinearities, but the one of most concern to soliton theory is self-phase modulation. With self-phase modulation, the optical pulse exhibits a phase shift induced by the intensity-dependent refractive index. The most intense regions of the pulse are slowed down the most, so they exhibit the greatest phase shift. Since a phase shift changes the distances between the peaks of an oscillating function, it also changes the oscillation frequency along the horizontal axis.

One may think of a phase shift as stretching out or squishing part of an oscillating function along its horizontal axis. This is easy to picture in terms of a sine curve, and the same principle applies for an optical pulse. Figure 3 shows a sine wave before undergoing a phase shift. This wave is unchirped, meaning that there is no ordered variation in frequency along the length of the wave. In the case of Figure 3, this is due to fact that the sine wave has a constant frequency along its entire length. Figure 4 shows the same sine wave after undergoing a phase shift such that the left-hand side of the wave has a higher frequency than the right-hand side. As a result, the wave is chirped—there is an ordered variation in frequency along the horizontal axis.

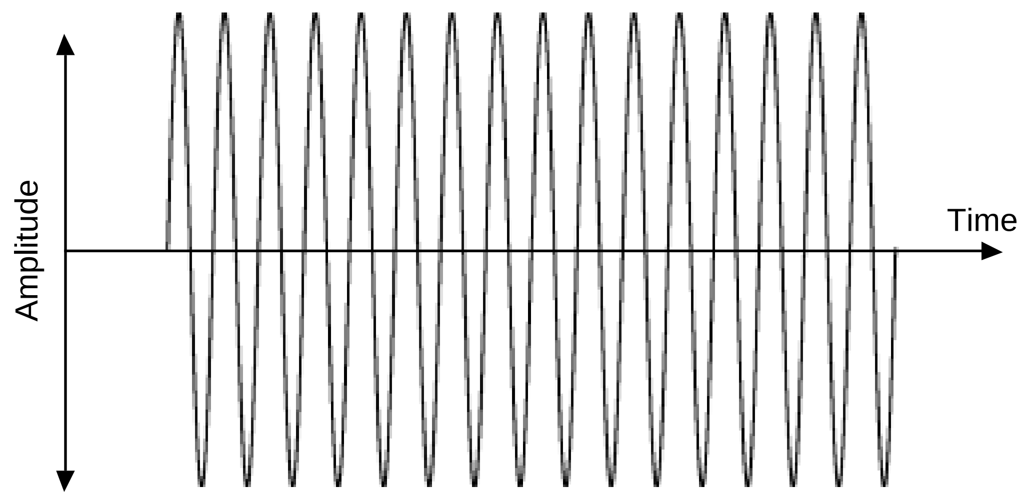

Figure 3   An unchirped sine wave

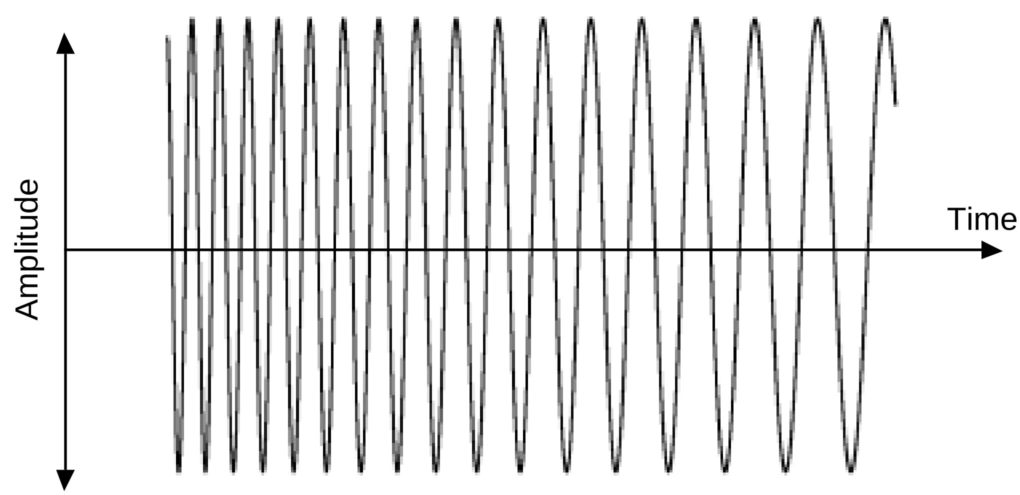

Figure 4   A chirped sine wave

The same concepts of phase shift and chirp may be applied to an optical pulse. Figure 5 shows an unchirped Gaussian pulse, and Figure 6 shows the same pulse after being chirped by a phase shift. (Please do not be confused about the difference in appearance between the Gaussian pulses in Figure 1 and Figure 5. Connect the peaks of the wave oscillations in Figure 5, and you will get a Gaussian curve like that in Figure 1. This simplification is known as a pulse envelope, and it is a common way of representing the shape of an optical pulse. Notice also that the chirped pulse in Figure 6 has the same envelope as the unchirped pulse in Figure 5. This is because self-phase modulation only broadens the pulse in the frequency domain, not the time domain.)

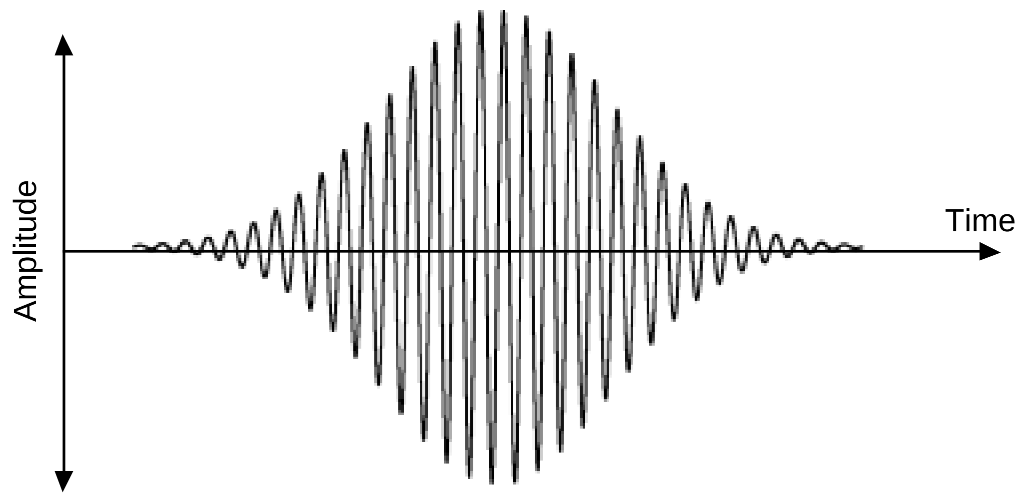

Figure 5 An unchirped Gaussian pulse

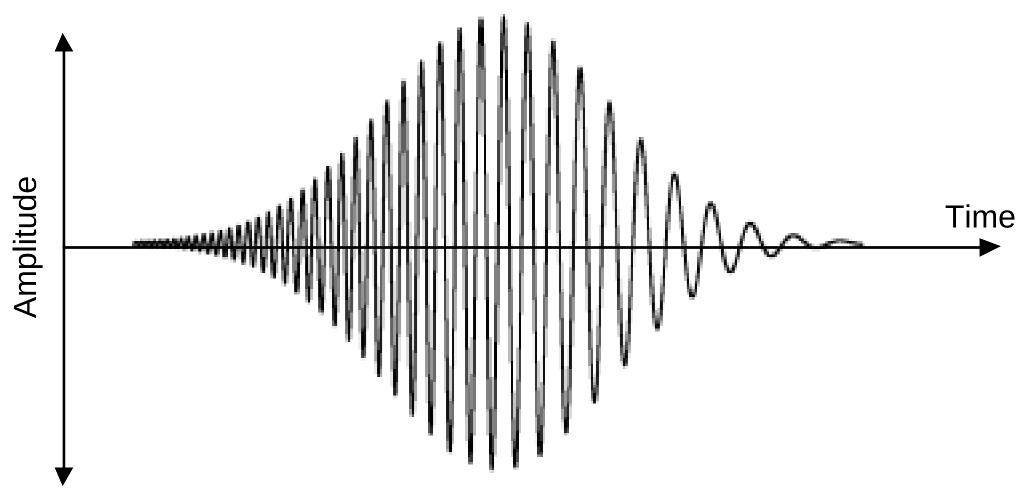

Figure 6 A chirped Gaussian pulse

As in Figure 6, self-phase modulation leads to a chirping with lower frequencies on the leading (right-hand) side and higher frequencies on the trailing (left-hand) side of the pulse. Like dispersion, self-phase modulation may lead to errors at the receiving end of a fiber optic communication system. This is particularly true for wavelength-division multiplexed systems, where the frequencies of individual signals need to stay within strict upper and lower bounds to avoid encroaching on the other signals.

The Formation of Solitons

We have now learned the origins of chromatic dispersion and self-phase modulation and how they separately affect the propagation of an optical pulse. Individually, these phenomena can create major problems in a fiber optic communication system, but if a system is designed such that the effects of dispersion and self-phase modulation perfectly cancel each other out, a pulse may propagate through a fiber without any broadening in the time or frequency domains. Let us now see how this is possible for a pulse with a wavelength in the anomalous dispersion regime of a fiber.

Self-phase modulation leads to lower frequencies at the leading side of the pulse and higher frequencies at the trailing side of the pulse. Anomalous dispersion causes lower frequencies to travel slower than higher frequencies. Therefore, anomalous dispersion causes the leading side of the pulse to travel slower than the trailing side, effectively compressing the pulse and undoing the frequency chirp induced by self-phase modulation. If the properties of the pulse are just right, the instantaneous effects of self-phase modulation and anomalous dispersion cancel each other out completely, and the pulse remains unchirped and retains its initial width along the entire length of the fiber. In other words, we have formed a soliton!

The formation of solitons requires a careful balance among the properties of the pulse and the fiber. Specifically, the following relationship must be satisfied:

$$P = -\frac{\beta_2}{\gamma T_0^2}, \tag{1}$$

where P is the peak power of the pulse, β_2 is the dispersion parameter (a negative value since it is in the anomalous regime of the fiber), γ is the nonlinear coefficient, and T_0 is the half-width of the pulse at its $1/e$ (36.79%) intensity point. The dispersion parameter and nonlinear coefficient depend on the properties of both the pulse and the fiber. They are given by

$$\beta_2 = \frac{\lambda^3}{2\pi c^2}\frac{\partial^2 n}{\partial \lambda^2} \text{ and } \gamma = \frac{2\pi n_2}{\lambda A_{eff}},$$

where λ is the pulse wavelength, c is the speed of light in a vacuum (3.0×10^8 meters per second), n is the refractive index, n_2 is the nonlinear index coefficient (a measure of the degree of fiber nonlinearity), and A_{eff} is the effective core area of the fiber. In addition, the pulse must be unchirped and have a hyperbolic-secant shape, as shown in Figure 7. If all of these qualifications are met, solitons may be successfully generated and propagated through a fiber.

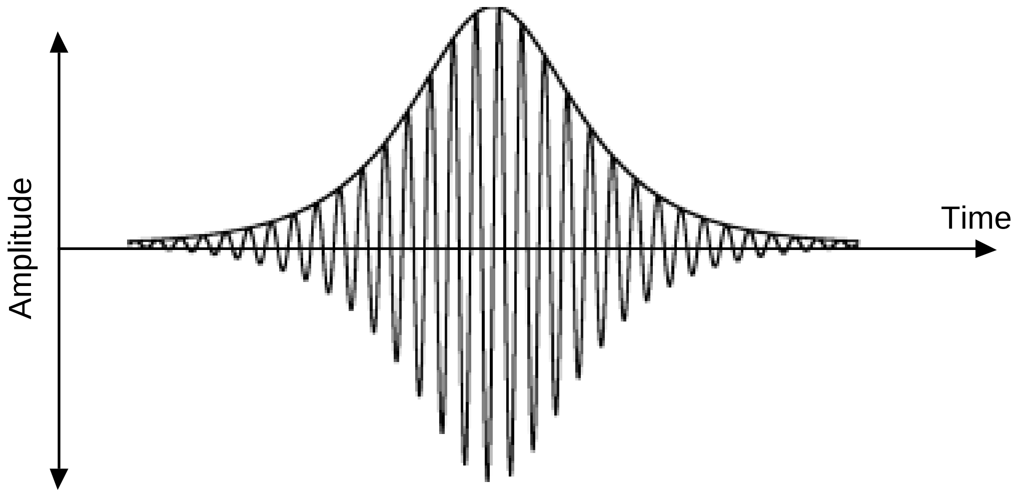

Figure 7 An optical soliton with a hyperbolic-secant envelope

But what happens if all of these criteria are not precisely met? This is an important question to consider since it is impossible in a real system to generate pulses with the exact parameters specified above for soliton formation. Theoretical and experimental results show that if the exact power and shape parameters are not satisfied, the interplay between dispersion and self-phase modulation can actually mold the pulse into a hyperbolic-secant shape with the appropriate peak power. Of course, the initial values must be close enough to the final soliton values in order for the soliton to form, and there are drawbacks when the input parameters vary substantially from the ideal soliton values. Namely, the dispersive effects will cause the pulse to lose some of its energy as it evolves into a soliton, and this energy loss can cause interference with neighboring pulses and therefore adversely affect the performance of a system.

Soliton formation is much more sensitive to chirp in the initial pulse than it is to inaccurate pulse shape and power. Ideally, the input pulse should be completely unchirped, but soliton formation may still occur is there is a very small amount of initial chirp. Again, the interplay between dispersion and self-phase modulation will eventually remedy the pulse, but it may lose a large percentage of its energy as it evolves into an unchirped soliton. The generation of unchirped pulses close to a hyperbolic-secant shape requires careful attention to the design of the semiconductor laser at the fiber input.

Issues with Soliton Propagation

Generating pulses with the appropriate parameters for soliton formation is just the first step in a successful soliton communication system: solitons must also be able to propagate through an optical fiber while maintaining these parameters. In particular, solitons must be able to retain their approximate peak power defined by Equation (1). This, of course, means that fiber loss is a huge potential problem.

Fiber loss occurs because the optical fiber is not 100% transparent. The pulse wavelength is chosen in order to minimize loss, but there is always some amount of absorption and scattering of the constituent photons. The result is a decrease in the power of the pulse, and if it is a soliton, the dispersion and self-phase modulation effects must broaden the pulse in order to maintain the correct relationship in Equation (1). Soliton broadening in soliton communication systems is harmful for the same reasons pulse broadening due to chromatic dispersion is harmful in non-soliton systems. This problem may be addressed by periodically passing the solitons through optical amplifiers, which counteract fiber loss by increasing the power of the pulse. The most common type of optical amplifier is the erbium-doped fiber amplifier (EDFA), which amplifies the pulse through a process of stimulated emission similar to what occurs in a laser. However, EDFAs and other amplifiers also introduce noise into the system via spontaneous emission, and this noise may perturb the system by interfering with the soliton signals and changing their parameters. Therefore, high-quality amplifier design is critical to the proper functioning of any soliton communication system.

The bit-rate of a soliton communication system is limited not only by the modulation speed of the laser used to generate the solitons, but also by the interactions that may occur between two neighboring solitons if they are too close together. These interactions, another result of fiber nonlinearity, may cause two adjacent solitons to periodically combine together and then separate from each other. If two neighboring solitons combine, they appear to the detector as just a single pulse; consequently, an error occurs. Soliton-soliton interactions may be minimized by ensuring that adjacent solitons are far enough apart that they do not interfere with each other. However, this may reduce the bit-rate of the system since it limits the number of solitons that may be transmitted in any given time interval.

Solitons may encounter many other problems or perturbations as they propagate through an optical fiber. Physicists and glass scientists must work carefully in order to avoid these potential problems and ensure accurate data transmission. This becomes especially complicated in wavelength-division multiplexed systems, where many signals travel through a fiber simultaneously but at slightly different wavelengths.

Closing Thoughts

We have already extolled the use of solitons in fiber optic communication systems, where they may be harnessed to provide highly accurate signal transmission over extremely long distances. But solitons may have their greatest impact on the development of photonic computers. Soliton switching has already been demonstrated by taking advantage of the solitons' resistance to pulse distortion. Furthermore, various logic gates (such as AND, XOR, NOR, and NOT) have been experimentally demonstrated using solitons in conjunction with other nonlinear optical phenomena like cross-phase modulation. These are all necessary components for the development of photonic computers, and with enough research and experimentation, they eventually will be combined with the appropriate interfaces to construct an all-optical computer chip.

Aren't you glad you learned about solitons?

En*light*ening Literature

We have just barely scratched the surface of optical soliton theory. If you would like to learn more, I strongly encourage you to check out the following literature.

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