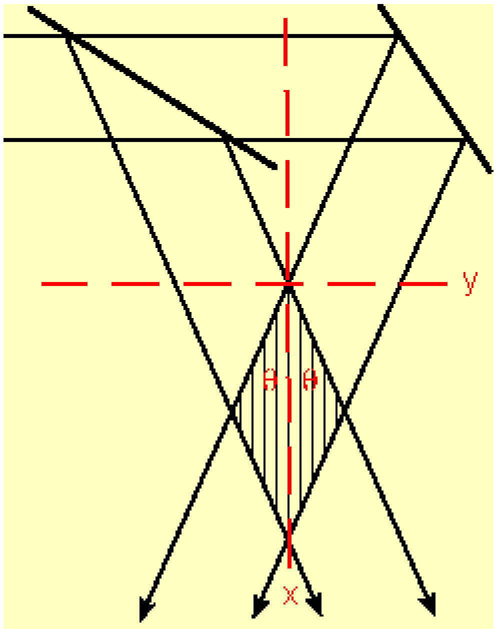
Interference of non-collinear beams.

Assume a laser beam ($\mathbf{E}(\mathbf{r},t) = \mathbf{E}_{max}\cos(\mathbf{k}\cdot\mathbf{r} - \omega t)$ with $k = 2\pi/\lambda$) is split into two beams that intersect as shown in the figure below.



$k_{1x} = k\cos\theta$, $k_{2x} = k\cos\theta$, $k_{1y} = k\sin\theta$, $k_{2y} = -k\sin\theta$.
In the regions where the beams overlap we have $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$.
Let $\mathbf{E} = E(\mathbf{z}/z)$.
$E(\mathbf{r},t) = A_1\cos(\mathbf{k}_1\cdot\mathbf{r} - \omega t) + A_2\cos(\mathbf{k}_2\cdot\mathbf{r} - \omega t)$.
$E(\mathbf{r},t)^2 = (A_1\cos(\mathbf{k}_1\cdot\mathbf{r} - \omega t))^2 + (A_2\cos(\mathbf{k}_2\cdot\mathbf{r} - \omega t))^2 + 2A_1\cos(\mathbf{k}_1\cdot\mathbf{r} - \omega t)A_2\cos(\mathbf{k}_2\cdot\mathbf{r} - \omega t)$.
$\cos A\cos B = (1/2)[\cos(A+B) + \cos(A-B)]$.
$\cos(\mathbf{k}_1\cdot\mathbf{r} - \omega t+\mathbf{k}_2\cdot\mathbf{r} - \omega t) = \cos(2k\cos\theta\, x - 2\omega t)$.
$\cos(\mathbf{k}_1\cdot\mathbf{r} - \omega t - \mathbf{k}_2\cdot\mathbf{r} + \omega t) = \cos(2k\sin\theta\, y)$.
$E(\mathbf{r},t)^2 = (A_1\cos(\mathbf{k}_1\cdot\mathbf{r} - \omega t))^2 + (A_2\cos(\mathbf{k}_2\cdot\mathbf{r} - \omega t))^2 + A_1A_2[\cos(2k\cos\theta\, x - 2\omega t)+\cos(2k_1\sin\theta\, y)]$.
$\langle I\rangle$ is proportional to $\langle\mathbf{E}(\mathbf{r},t)^2\rangle$.
The average values of $\cos^2(\mathbf{k}_1\cdot\mathbf{r} - \omega t)$ and $\cos^2(\mathbf{k}_2\cdot\mathbf{r} - \omega t)$ are ½, and $\cos(2k\cos\theta\, x - 2\omega t)$ is zero, when averaged over a large number of periods.
$\langle\mathbf{E}(\mathbf{r},t)^2\rangle = A_1^2/2 +A_2^2/2 + A_1A_2\cos(2k\sin\theta\, y)]$.
$\langle I\rangle = \langle I_1\rangle + \langle I_2\rangle + 2(\langle I_1\rangle\langle I_2\rangle)^{1/2}\cos(2k\sin\theta\, y)]$.
The intensity therefore varies with y as $C_1 + C_2\cos(2k\sin\theta\, y) = C_1 + C_2\cos((2\pi/\lambda')y)$, $\lambda' = \pi/k\sin\theta$.
We observe fringes with fringe spacing λ'.